SYNOPTIC: Boundary-Layer Computation by an N Parameter Integral Method Using Exponentials, Hartmut H. Bossel, Mechanical Engineering Department, University of California, Santa Barbara, Calif.; *AIAA Journal*, Vol. 8, No. 10, pp. 1841–1845.

Boundary Layers and Convective Heat Transfer

Theme

Describes an integral method (method of weighted residuals) which uses exponentials in approximating and weighting functions and an arbitrary number N of parameters. Method is used to solve plane laminar incompressible boundary-layer equations. It is applied to several standard test cases and results are compared with accepted solutions.

Content

Let X = nondimensional streamwise coordinate; Y = nondimensional coordinate normal to X; U = nondimensional velocity in X direction; U_e = velocity at the edge of the boundary layer (prescribed); $U_{\text{tab}}(Y)$ = tabulated experimental or analytical initial velocity profile; V = nondimensional velocity in Y direction; V_0 = nondimensional wall suction or blowing velocity (prescribed); $f_k(Y)$ = weighting functions, finite everywhere with $f_k(\infty) = 0$; $a_n(X)$ = unknown parameters in the velocity approximation; and α = prescribed exponent, here $\alpha = 1$.

The nondimensional laminar incompressible boundary-layer momentum equation,

$$\partial U^2/\partial X + \partial(VU)/\partial Y = U_e(dU_e/dX) + \partial^2 U/\partial Y^2$$

is multiplied by weighting functions $f_k(Y)$ and integrated to

$$\frac{d}{dx}\int_0^\infty f_k U^2 dY - \int_0^\infty f_k' VU dY - \int_0^\infty f_k'' U dY - U_e \frac{dU_e}{dX}\int_0^\infty f_k dY + \left[f_k \frac{\partial U}{\partial Y}\right]_{Y=0} = 0 \quad (1)$$

$$k = 1, 2, \ldots, N$$

where

$$V(X,Y) = -\int_0^Y \frac{\partial U}{\partial X}\, dY + V_0(X)$$

(The paper employs a minor transformation not shown here.) Choice of weighting functions,

$$f_k(Y) = e^{-kY} \qquad k = 1, 2, \ldots, N$$

and velocity approximation,

$$U(X,Y) = (1 - e^{-\alpha Y})[U_e(X) + \sum_{n=1}^{N} a_n(X)e^{-n\alpha Y}] \quad (2)$$

permits formal integration of Eq. (1) with respect to Y. A system of N ordinary differential equations for N parameters $a_n(X)$ remains and is solved by standard numerical methods. The velocity profile follows from Eq. (2); shear stress, displacement, and momentum thicknesses follow from similar formulas.

Parameters for the initial profile are obtained from either a set of algebraic equations to which the ordinary differential equations reduce in the case of similarity, or from a more general least-squares procedure by requiring

$$\int_0^\infty f_k U(a_n; Y) dY = \int_0^\infty f_k U_{\text{tab}}(Y) \equiv q(k)$$

$$k = 1, 2, \ldots, N$$

where the $q(k)$ are obtained by numerical integration. The N equations yield N initial profile parameters a_n.

The integral method is used to compute several standard boundary-layer test cases covering the spectrum of boundary-layer flows from stagnation point to separation: 1) discontinuous external pressure gradient (stagnation point flow changing into flat plate flow); 2) discontinuous suction applied to flat plate flow; and 3) circular cylinder with suction. The results are compared with accepted published solutions. Good qualitative agreement is found for N = 1 or 2, while very accurate results are obtained for N = 3–5. For N = 5 the absolute error in the wall shear is generally of order 10^{-4}.